

Selecting alternatives in the conceptual design phase: an application of Fuzzy-AHP and Pugh's Controlled Convergence

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Abstract The selection of conceptual design alternatives is crucial in product development. This is due both to the fact that an iterative process is required to solve the problem and that communication among design team members should be optimized. In addition, several design constraints need to be respected. Although the literature offers several alternative selection methods, to date, only very few are currently being used in industry. A comparison of the various approaches would improve the knowledge transfer between design research and practice, helping practitioners to approach these decision support tools more effectively. This paper proposes a structured comparison of two decision support methods, namely the Fuzzy-Analytic Hierarchy Process and Pugh's Controlled Convergence. From the literature debate regarding selection methods, four relevant criteria are identified: computational effort, suitability for the early design stages, suitability for group decision making, and ease of application. Finally a sensitivity analysis is proposed to

test the robustness of each method. An industrial case study is described regarding an innovative and low-cost solution to increase the duration of heel tips in women's shoes. The selection of conceptual design alternatives of the heel tip presents complex challenges because of the extremely difficult geometric constraints and demanding design criteria.

Keywords Engineering design methods · Concept selection · Fuzzy-analytic hierarchy process · Pugh's controlled convergence

1 Introduction

Early product introduction and cost reductions in the manufacturing process are fundamental for a successful industrial product. In the literature it has been reported that 75 % of the cost of a product is due to the design phase [1] and 80 % to the conceptual design process [1,2]. Moreover, a misconceived conceptual design can never be compensated for by a later better detailed design [3]. Thus, money and time should be invested wisely in the early design stages and, in particular in the conceptual design phase. Tailor-made methods and CAD-based tools are often developed by engineers to reduce errors and time wasting in these delicate phases [4,5].

The concept design process is made of steps, characterized by strict coordination and collaboration [6]. Thus, the customer's needs have to be evaluated first, followed by the target specifications and the product's particular requirements.

Concept selection is a complex task for engineering designers as it can be considered as the most critical decision-making step in the product development process [7]. During this phase, erroneous solutions need to be minimized, which means that several facets of the problem have to be considered concurrently.

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Efficient design methods and suitable decision-making techniques thus both contribute to a successful final product.

Multicriteria decision-making methods are effective tools for selecting preliminary designs throughout the product development process. According to Vincke [8], multicriteria decision aids provide the decision maker with problem-solving tools, in the case of multiple and conflicting criteria.

Montagna [9], for example, suggests using Knowledge Management methods for the collaborative aspects of the conceptual design phase, Systematic methods (such as Pugh's Controlled Convergence-PuCC) for the concept selection phase, and Multicriteria Decision Making methods (such as the Analytic Hierarchy Process-AHP) for the evaluation of concept design alternatives.

Sharing these premises, Kuppuraju et al. [10] define the engineering design process as a problem of selection or the improvement of an alternative. They describe the selection process in two phases. The first consists in generating feasible concepts and the second involves the selection-Decision Support Problem (DSP).

This is crucial for designers, because they have to choose the alternatives that will be developed in a further detailed stage.

However, as highlighted by Salonen and Perttula [11], little use of concept selection methods is made. Yang [12] reports that, out of 106 experienced engineers, only a small fraction (15 %) employ concept selection methods and even less (13 % of the 15 %) are satisfied with the results.

There may be several reasons for such diffidence. One lies in the fact that industrial experts tend to rely on their experience and know-how. This is partially due to the fact that although there are several selection applications in the literature, few studies focus on comparisons between the methods.

This work provides a structured comparison between two well-known design selection methods, highlighting the advantages and disadvantages of the methods. In this way the paper aims to contribute to the knowledge transfer between design research and industrial application.

The comparison is made between the following different representative approaches: the Fuzzy Analytic Hierarchy Process (F-AHP) and Pugh's Controlled Convergence (PuCC).

F-AHP, or AHP in Saaty's original version [13], is structured analytically. On the other hand, PuCC, is characterized by a more discursive framework, in which team discussions are required in order to provide a unique group preference.

A list of common criteria were proposed for the comparison and a sensitivity analysis was performed to ensure the robustness of each application. The analysis was also carried out to assess the dependence of the results on small changes in the preference weights for the criteria and/or on the rating values of the alternatives.

A case study on the conceptual design of heel tips for women's shoes is proposed, to better illustrate and compare the two selection methods.

1.1 Related research

1.1.1 F-AHP in concept selection

The complexity characterizing F-AHP and other decision making methods may be one of the reasons why they are not very common in industry. In fact, AHP-based methods involve several mathematical calculations, increasing with the number of criteria and alternatives.

This is clear from Buyukozkan et al.'s table [14] (proposed again in [15]), in which the advantages and the drawbacks of the application of F-AHP-based methods are described. The most cited advantage of this selection method is its ability to focus on the perspective of the decision makers. In addition, the F-AHP process is suitable for software implementation.

The F-AHP phases are however more complex than the three iterations traced by the PuCC's matrices. This aspect was discussed by Stuart Pugh, who argued that, during the early stages of the design process, such a high level of precision is not required [16].

The Fuzzy version of the AHP method has nevertheless been used to overcome some limitations observed in the application of Saaty's original version. These weaknesses have already been described in the literature. Ayag [17] for example, remarked that AHP is mainly used for crisp decision making. In fact, it does not consider the uncertainty associated with the loss of information, due to the translation of the verbal judgments of the decision makers into crisp numbers. Moreover, the decision maker's requirements may contain ambiguity and the human judgment on quality attributes may be imprecise. Thus, the crisp aspect of the conventional AHP seems inappropriate in depicting the uncertain nature of this decision phase. A Fuzzy-AHP selection approach is considered as a remedy to these drawbacks.

Several examples in the literature regard the use of AHP techniques to rank alternatives and solve product development sub-problems. The most commonly described fields of application are: product family evaluation, selection of manufacturing technologies, and robot or equipment selection for a specific industrial purposes.

AHP is often combined with other intelligent techniques. Combined with the fuzzy sets theory, AHP is used to consider uncertainties during the early stages of design and deal with the variables in verbal judgments. Fuzzy interfaces to transform the linguistic variables into fuzzy numbers are often used to avoid the uncertainty brought by numerical voting.

Liu et al. [18] introduced a fuzzy decision making method to select the optimum design, using conjoint analysis to consider both customer preferences and engineering constraints.

In [19] the authors propose a Fuzzy-Interface/AHP-based software application for the selection problem, with a Fuzzy conversion of input/output linguistic variables and a sub-criteria dependence approach.

Another fuzzy selection procedure is proposed by Liang and Wang [20], who, after classifying the product attributes into subjective and objective ones, translate the verbal judgments of decision makers when weighing up attributes against alternatives, into fuzzy variables, in order to avoid problem constraints.

1.1.2 PuCC in concept selection

In [16], pairwise comparison strategies and Pugh's method in particular, were tackled, by analysing [21], to highlighting the advantages and disadvantages of the pairwise comparison methodology. According to these authors, pairwise-based techniques may introduce errors in the decision-making process and, consequently, lead to erroneous conclusions. Moreover, important information regarding the intensity of comparisons may be lost, thus reducing the comparison to a simple preference estimation. The same authors describe the clarifications made in [22], regarding the capabilities that a concept evaluation method should have.

One interesting attempt to enhance Pugh's Controlled Convergence method consists in adding an evaluation of the probability of the alternatives satisfying the target requirements. Frey et al. [23] consider this approach appropriate for cases where there are few alternatives with well-known characteristics, without any possibilities of generating new alternatives. The authors therefore propose a model-based evaluation for quantitatively performing the PuCC method. In fact, while in the PuCC method, there is no quantitative technique for eliminating concepts, the authors propose a direct elimination of the dominated concepts. They thus simulate the elimination of concepts, in order to understand how the strength of a datum concept influences the convergence of the set of alternatives. The simulation is carried out by means of four cases, known in the literature, in three of which only one datum concept is considered. The analysis revealed that the selection of a strong datum concept substantially reduces the alternatives at the first run (25–70 % reduction in the number of alternatives). If the datum concept is not so strong on the other hand, it results in a lack of convergence at the first run.

Frey et al.'s experiment also showed that, due to the iterative nature of the process, a single run almost never leads to one unique solution.

Several applications deal with the use of the PuCC method in product concept selection. For instance, in [24], the design of an improved impulse turbine is proposed, combining the Pugh concept analysis and 3D CAD design.

Iqbal et al. [25] use Pugh's Controlled Convergence method to generate a wide variety of design concepts of

a wing for a Medium Altitude Long Endurance (MALE) Unmanned Aerial Vehicle within a CAD environment, applying and iterating the process until the best concept is selected.

Wang [26] extends Pugh's concept selection method with a fuzzy set theory, in order to provide a measure of the quality of a chosen concept, by supporting designers with numerical information.

1.2 Comparison strategy

1.2.1 Comparison criteria

The comparison between the F-AHP and PuCC methods was carried out by selecting the four criteria outlined below which have been derived from the literature.

1. Computational effort required.

The computational effort is a measure of the time needed to introduce data and implement the method. It is widely considered that decision support problems are not common in engineering design practice [11, 27–30]. Thus, an indication of the computing time and effort could be interesting in order to encourage the use of such techniques in engineering design practice.

2. Suitability for the early design phases.

Early design phases and, in particular, the conceptual phase, are riddled with uncertainties and lack of information. The fact that analytical techniques may be unsuitable for the early design phases has been reported in the literature [16].

3. Suitability for group decision making.

The suitability for group decision making reveals how the method takes into account the preferences of an individual decision maker. This is a measure of capability in decision-making, among several members. This is a fundamental aspect in tackling engineering design problems, where discussion between members is paramount for a suitable trade-off regarding product requirements.

4. Ease of application

This criterion is also fundamental in the choice of a decision support method. In fact, in our experience in the industrial field, one of the main reasons for rejecting a method is because it is difficult to apply. Users are likely to prefer a decision support method that only has few instructions.

On the other hand, more analytically complex methods, such as F-AHP or TOPSIS, are usually more suitable for software implementation, due to their "analytical" nature. There-

fore, if a simple interface for simplifying data input could be built and proposed to the user, some apparently complex methods would probably be more attractive for practitioners than “less analytic” ones.

In this paper, we tackle this apparent contradiction, to clarify both the strengths and limitations of the two chosen methods.

1.2.2 Work flow of selection methods

F-AHP The F-AHP method originated from Saaty’s AHP [13] and, like the latter, is based on a hierarchical structure. On the top level is the main goal to be attained; on the second level are the given criteria, both quantitative and qualitative, and any other sub-criteria are on the levels below.

The alternatives are located at the bottom of the hierarchy.

The method is based on pairwise comparisons, in the sense that the criteria are first compared pairwise with respect to the main goal so as to put the criteria in order of priority. Thus, the alternatives are also pairwise compared, with respect to the criteria. The aim here is to produce a classification, which reflects the degree of importance given to each criterion, with respect to the main goal. Finally, a concordance index offers a measure of the consistency and reliability of the results.

In F-AHP, the linguistic variables used for the verbal judgments are translated into Fuzzy numbers. This approach has been reported in the literature (e.g. [18,19]). A numerical 9-point scale, such as the one proposed by Saaty [13], is simple and suitable for crisp judgments, however it does not properly describe the uncertainty of translating a human perception into a number [15].

For the case study proposed, the linguistic assessments of the decision makers are described using Trapezoidal Fuzzy Numbers. As remarked in [19], a Fuzzy Number is described by its membership function $\mu_A(x)$ with the following characteristics:

- $\mu_A(x) = 0, \forall x \in (-\infty, \alpha] \cup [\delta, \infty)$;
- $\mu_A(x)$ increases monotonically in $[\alpha, \beta]$ and decreases in $[\gamma, \delta]$
- $\mu_A(x) = 1, \forall x \in [\beta, \gamma]$;

or it may also take these values: $\alpha = -\infty$ or $\alpha = \beta$ or $\beta = \gamma$ or $\delta = \infty$. The fuzzy number is described by a 4-tuple $[\alpha, \beta, \gamma, \delta]$, with straight line segments for $\mu_A(x)$ in $[\alpha, \beta]$ and $[\gamma, \delta]$.

From a mathematical viewpoint, a Trapezoidal Fuzzy Number has a membership function: $\mu_A : \mathbf{R} \rightarrow [0, 1]$ as in Eq. (1) and depicted in Fig. 1.

$$\mu_A(x) = \begin{cases} (x - a)/(b - a) & \text{for } a \leq x \leq b \\ 1 & \text{for } b \leq x \leq c \\ (x - c)/(d - c) & \text{for } c \leq x \leq d \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

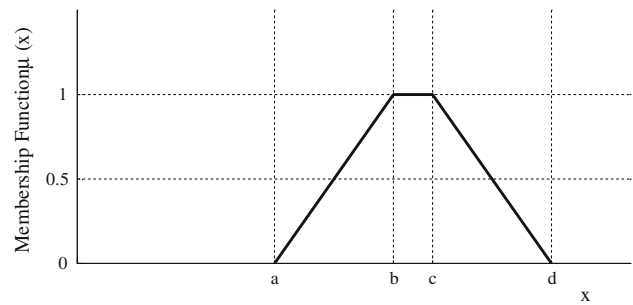


Fig. 1 Trapezoidal membership function

The F-AHP method, as described in Fig. 2, is based on the following steps. First, the decision makers express their preference by pairwise comparing the given criteria: these linguistic assessments are handled using fuzzy numbers.

Next the criteria are put in order of priority and a matrix with the values of the mean aggregated preference weights is thus provided.

The suitability of each alternative is also evaluated, with respect to the criteria, by means of linguistic variables, treated as Trapezoidal Fuzzy Numbers.

Finally, consequent fuzzy score evaluation and a defuzzification procedure is carried out to generate a ranking vector of the alternatives. A software tool based on the algorithmic form of the F-AHP process was implemented in order to simplify usability for industrial purposes.

The mathematical procedure that enables the F-AHP to evaluate the design alternatives is described below.

In the first step the decision makers provide their preferences for the criteria, using verbal judgments taken from the set “preference”. The weights are allocated for each criterion j , ($j = 1, \dots, i$), by the r th decision maker ($r = 1, \dots, q$). In this way, a matrix of $(i \times q)$ elements is produced, as in Eq. (2).

$$(w_{jr}) = \begin{pmatrix} w_{11} & w_{12} & \dots & \dots & w_{1q} \\ \dots & \dots & & & \\ & & \dots & & \\ w_{i1} & \dots & \dots & \dots & w_{iq} \end{pmatrix} \quad (2)$$

Each weight w_{jr} is a four-element vector, representing the trapezoidal number $w_{jr} = (a_{jr}, b_{jr}, c_{jr}, d_{jr})$.

The linguistic values provided by the decision makers to evaluate the suitability of each alternative in relation to the criteria, are translated into the corresponding numerical values, in a $(i \times q \times 4)$ matrix (as each element is a four-element vector).

Hence, the Mean Operator is used to find the *mean aggregated preference weight* W_j in Eq. (3), thus generating a $(i \times 4)$ -sized W_j matrix.

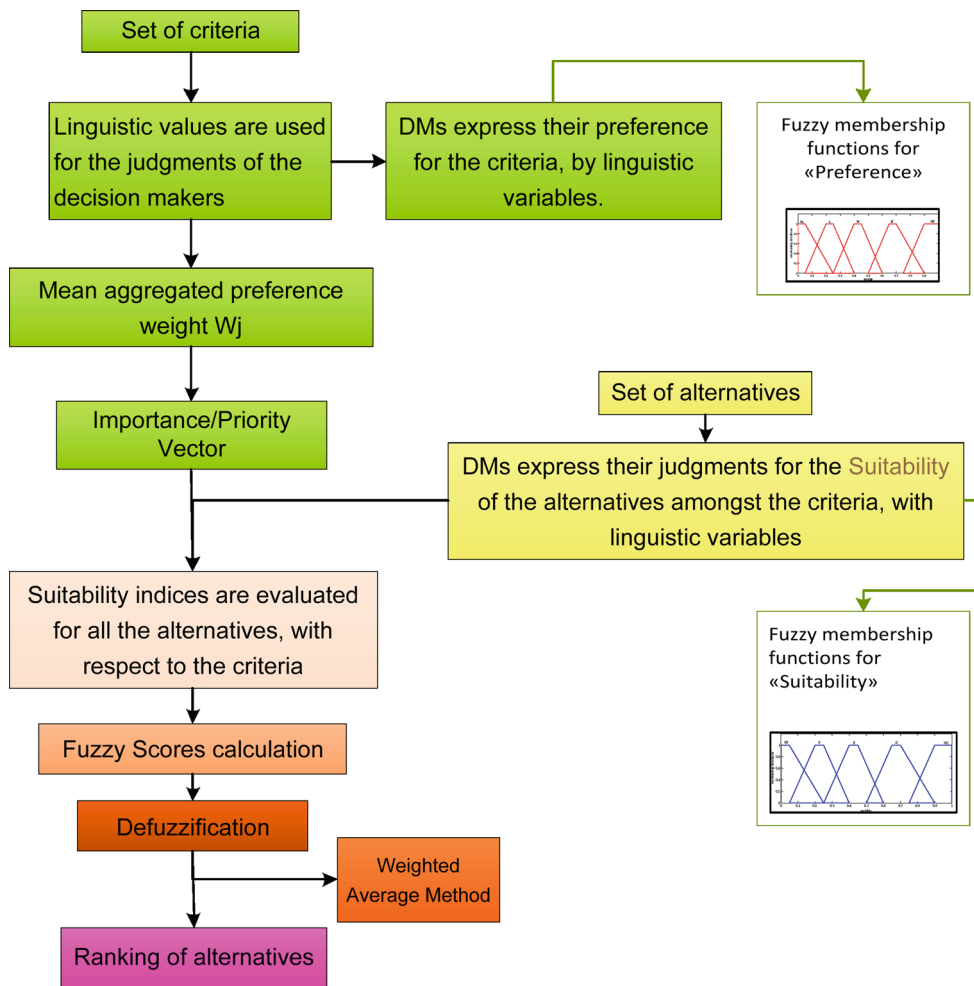


Fig. 2 Fuzzy-AHP procedure

$$W_j = \frac{\sum_{r=1}^q w_{jr}}{q} = \left(\frac{\sum_{r=1}^q a_{jr}}{q}, \frac{\sum_{r=1}^q b_{jr}}{q}, \frac{\sum_{r=1}^q c_{jr}}{q}, \frac{\sum_{r=1}^q d_{jr}}{q} \right) \quad (3)$$

Therefore, the q decision makers are again required to provide judgments on the suitability of each alternative, with respect to the criteria, using the linguistic variables from the Term Set of “suitability”.

If $S_{jmr} = (a_{jmr}, b_{jmr}, c_{jmr}, d_{jmr})$ is defined as the suitability assigned to the m th alternative ($m = 1, 2, \dots, p$), evaluated against the j th criterion ($j = 1, 2, \dots, i$), by the r th decision maker ($r = 1, 2, \dots, q$), the suitability is given by i matrices of size $(p \times q)$.

The suitability indices (S_{jm}) are then evaluated for each criterion, as in Eq. (4)

$$S_{jm} = \frac{\sum_{r=1}^q S_{jmr}}{q} \quad (4)$$

The matrices of the suitability indices can be collected in a single matrix that has the form of Eq. (5) with i criteria and p alternatives.

$$S_{indices_m,j} = \begin{bmatrix} S_{1j1} & S_{1j2} & S_{1j3} & S_{1j4} \\ \vdots & \vdots & \vdots & \vdots \\ S_{mj1} & S_{mj2} & S_{mj3} & S_{mj4} \\ \vdots & \vdots & \vdots & \vdots \\ S_{pj1} & S_{pj2} & S_{pj3} & S_{pj4} \end{bmatrix} \quad (5)$$

In order to proceed with the final ranking, a Fuzzy Score Γ is calculated as in Eq. (6), for each alternative $m = 1, \dots, p$.

$$\Gamma_m = \frac{\sum_{j=1}^i (S_{jm} \otimes W_j)}{i} = \frac{(S_{1m} \otimes W_1) \oplus \dots \oplus (S_{jm} \otimes W_j) \oplus \dots \oplus (S_{im} \otimes W_i)}{i} \quad (6)$$

The matrix corresponding to the evaluation of the fuzzy scores, for all the alternatives, is shown in Eq. (7).

$$\Gamma_{tot} \equiv \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \\ \dots \\ \Gamma_m \\ \dots \\ \Gamma_p \end{pmatrix} \tag{7}$$

A final defuzzification is performed, in which the Fuzzy Score of each alternative is changed into an equivalent crisp value. This is done using the Weighted Average Method [as shown in Eq. (8) for each alternative] [20]. Each of the base variables x is weighted by its respective maximum membership value $\mu_m(x)$ for any m Fuzzy Number (or Variable).

$$\Gamma_{mean} = \frac{\sum (\mu_m(x) \times x)}{\sum \mu_m(x)} \tag{8}$$

The defuzzified values are thus collected in a vector, in which each row corresponds to the mean value of the fuzzy score of each alternative. This vector provides a classification of alternatives, in the sense that the highest score corresponds to the best alternative.

PuCC There are two phases in Pugh’s Controlled Convergence method: screening and scoring. Ulrich and Eppinger [6] characterize these two phases using the “screening” and the “scoring” matrices.

In the first phase no vote is given, as alternatives are screened in relation to the criteria. The alternatives are simply defined as being “better” (+), “equal” (0) or “worse” (–), than an alternative taken as a comparison, called “*concept datum*”.

The second phase regards the selection decision support problem, (Decision Support Problem-DSP [10]), in which, the relative importance of the criteria is evaluated and the feasible alternatives are ranked for each criterion.

As depicted in Fig. 3, this phase includes the ranking of the criteria and the rating of the alternatives, with respect to the criteria. The alternatives are ranked using a voting procedure and taking account of the priority of the criteria.

The process is iterative: at each run, the alternatives with the minimum score are eliminated. Several iterations can be performed, each with a different datum concept. In the second run, the best-scoring alternative of the previous run is selected as the new concept datum.

In the scoring phase, first the criteria are pairwise compared, so that *one* point is awarded to the preferred alternative and *none* to the worst. Half a point indicates an equal preference between the criteria.

A total net rating is thus calculated for each criterion and the corresponding normalized values identify the elements of the importance vector—“priority vector”—for the criteria. Each element of the priority vector is defined as the ratio of the rating of each criterion and the sum of the total ratings of the criteria.

The alternatives selected from previous iterations, designated as candidates for the selection process, are thus rated against the given criteria. A maximum score is defined for each alternative.

The normalized values R_{ij} of the alternative ratings A_{ij} are calculated as the ratio of the alternative rating and the total sum of the maximum possible score of the corresponding alternative [Eq. (9)].

$$R_{ij} = \frac{A_{ij}}{A_{ijMax}} \tag{9}$$

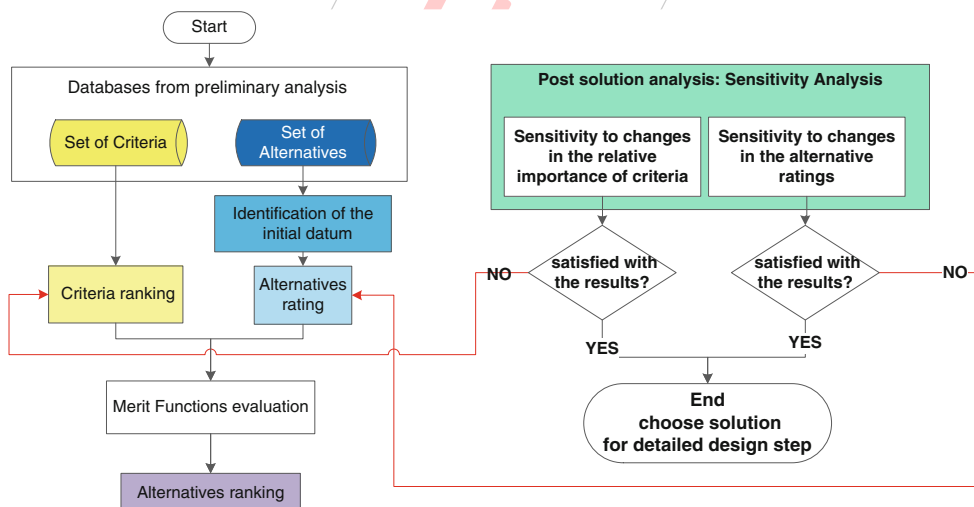


Fig. 3 The DSP for the PuCC method and the post-solution sensitivity analysis

The merit functions MF_i , which are used to rank the alternatives, are calculated for the i -th alternative using the linear additive form of Eq. (10). In fact this mathematical form is straightforward and is the most commonly used [10].

$$MF_i = \sum_{j=1}^n I_j \cdot R_{ij} \quad i = 1, \dots, m \quad (10)$$

where R_{ij} is the normalized rating for alternative i with respect to criterion j [Eq. (9)].

1.2.3 Sensitivity analysis

To test the robustness and stability of the solution candidates, a post-solution sensitivity analysis is performed by changing either the attribute weights or altering the alternative ratings.

This analysis can be performed for each method independently and not necessarily in a comparable way. Different ways of evaluating the weights of the criteria in the two selection methods would lead to different approaches for the sensitivity analyses. In fact, while in the F-AHP, verbal judgments are provided for the weights, in the PuCC method, numerical votes are allocated to them.

Therefore, for the sensitivity analyses we considered two quite different approaches [31] for F-AHP and [10] for PuCC. In the first approach, the verbal judgments are forced to be all equal to one judgment at a time. In the second, small changes are made to the numerical votes allocated to the criteria.

2 Case study: selection of conceptual design of heel tips for women's shoes

The case study used to enable us to compare the F-AHP and PuCC methods was provided by an Italian heel manufacturer. The problem consisted in finding an innovative and low-cost method to increase the duration of heel tips in women's shoes.

This issue involves multiple constraints and thus, can be treated as an MCDM (Multi Criteria Decision Making) problem. The overall objective of the research is to increase the life of the heel-tip for women's shoes, which is subject to wear due to contact with the ground. The polymeric layer of the heel-tip tends to wear prematurely. This problem generally occurs within a few days of continual use in high heels shoes with a reduced cross-section.

When the polymer wears down this exposes the head of the metal pin which also generates a noise during walking, due to the contact with the ground. This is perceived as a problem by the customers.

Specifications for developing new solutions for heel to heel-tip connection, mandate that the original shape and overall dimensions be preserved.

There are strict constraints regarding the interface between the innovative heel tip and the heel structure. In fact, an elastic

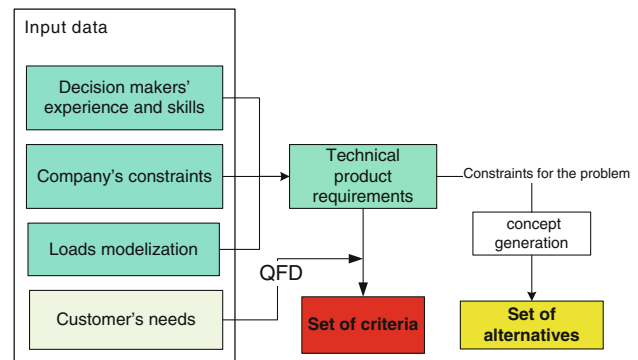


Fig. 4 Generation of the sets of criteria and alternatives, for the multicriteria decision making problem

metal pin is inserted into the plastic heel for reinforcement. The internal dimension of the metallic pin is less than 3 mm. The pin of heel tips currently on the market is forced into the pin hole.

In addition, the heel to heel-tip connection needs to fit the whole range of high heel women's shoes. The conflicting constraints of the compact size and shape, together with stiffness values need to be respected concurrently, which makes finding a solution all the more difficult. Experts from several disciplines tackled the problem since the early stages of the design process. In fact, at this initial design stage, several uncertainties need to be considered due to the lack of information regarding the relevant data.

This is in agreement with other research [32].

The concept selection followed in our specific experience, covered the steps shown in Fig. 4: first the input data need to be considered for the problem. Decision makers have to provide their experience and know how in order to identify the most relevant requirements for the product. These requirements form part of the product criteria, while other criteria include the constraints provided by the manufacturer of the heels, and the customer needs.

Firstly, a meeting is held between three decision makers—the manufacturer, the engineering designer, and a market expert in heel manufacturing together with a few customers. Working together they list the product specifications,.

As often occurs in multicriteria engineering, conflicting constraints need to be handled and considered concurrently. In fact, while the manufacturer prioritized the manufacturability and the reliability of the mechanical assembly, the expert in heel production stressed the importance of innovation. The engineering designer focused on increasing product stiffness by modifying the material for the heel-tip and pin, with a consequent change in the technological production process.

A Quality Function Deployment was then applied to transform the needs into technical requirements and specifications.

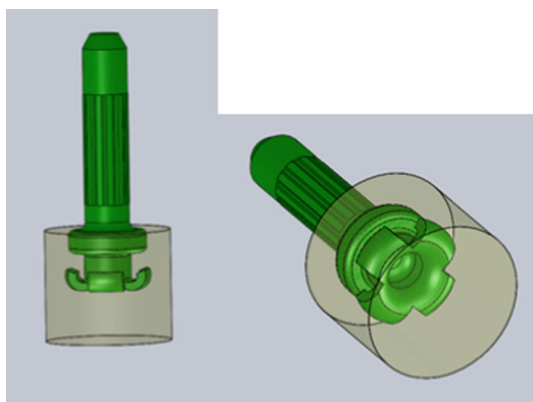


Fig. 5 Commercial heel-tip used in the case study

Two different approaches were proposed to solve the design problem: to increase the duration of the material or to increase the height of the heel tip, using an innovative design for the metal pin.

Thus starting from the international standards regarding the stresses acting on the foot during walking, a preliminary mathematical model was built and validated by FEM analysis, for a rough calculation of the loads and constraints acting on the commercial heel-tip shown in Fig. 5.

2.1 The decision-making experiment

The F-AHP method was applied first and the decision makers were required to vote separately without sharing their choices. Precautions were taken to avoid biases due to the repetition of the same experiment with a different tool. First, the F-AHP method was applied before the PuCC as previous discussions between team members preceding Pugh's decision process, could have affected the results. Secondly, team members were not allowed to communicate with each other before the completion of the F-AHP voting phase.

2.1.1 Assumptions

The first condition for the problem is that decision-makers do not change their minds between one experiment and another. This condition is very rigid, because no control can be provided for rationalizing changes in preference within a certain time interval.

This is most evident in group discussions, in which opinions may be influenced by other members and thus change considerably during the decision-making process. This issue was tackled in an experiment by Ji et al. [33] in which a team discussion was divided into intervals, and the designer's preferences were assumed to remain unchanged. In addition, a distinction was made between the formal and informal aggregation of preferences. In fact, a consensus for the entire group is unlikely to be reached in a formal way. More informal ways

Table 1 Selected criteria for the analysis

Criteria	
C1	Reduction of the pin height (add volume for wear material)
C2	Aesthetics
C3	Assembly reliability
C4	Manufacturability
C5	Development cost
C6	Minimal modification to the heel assembly

on the other hand, such as voting or consensus building, are thought to be effective for assigning group preferences.

In our case study, however, we assumed that the decision makers do not change their minds regarding their preferences from one session to the other. This hypothesis is supported by the fact that similar final results emerge from the application of the two methods.

Another important consideration is the fact that the two approaches are based on different voting rules, which could affect the final results. In fact, while in PuCC a team discussion leads to total consensus, in F-AHP each decision maker expresses their own judgment irrespective of the others. Thus, members are not influenced by each other in the F-AHP voting procedure and thus bias is avoided.

After the definition of the case study, the frameworks of F-AHP and PuCC were developed, implemented in spreadsheets and then applied.

The criteria were listed, starting with the requirements highlighted by the experts. Six criteria were highlighted, as reported in Table 1.

The first criterion (C1) regards the possibility of reducing the height of the heel-tip pin, in order to enable other polymeric material to be used. C2 (aesthetics) is critical in terms of a successful market introduction. C3 (assembly reliability) can be measured by the area involved in the gripping of the polymeric material by the heel of the heel-tip pin. C4 (manufacturability) measures the ease of implementing the technology. C5 (development cost) is related to the cost of the manufacturing process. To be more precise, the analysis of costs should follow the decision-making process, at a later stage. In this context, however, the cost was considered as a criterion in itself, to simplify the comparison between the selection methods.

The last criterion (C6) is the minimal modification made to the heel assembly, which takes into account the geometrical constraints of the assembly.

Following these constraints, six innovative concept designs were thus proposed (Fig. 6). To better understand the complexity of the problem and the variety of solutions provided, we now describe the main characteristics of the heel assembly and the alternatives. The heel-tip device and the plastic heel are assembled by press fitting the metallic pin of the heel

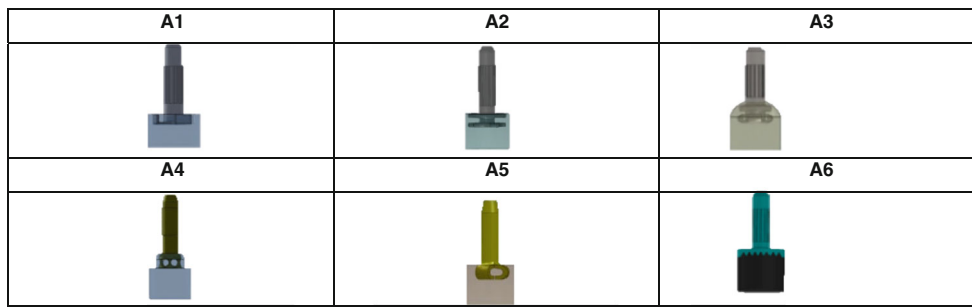


Fig. 6 Innovative conceptual design alternatives, proposed for the heel-tip

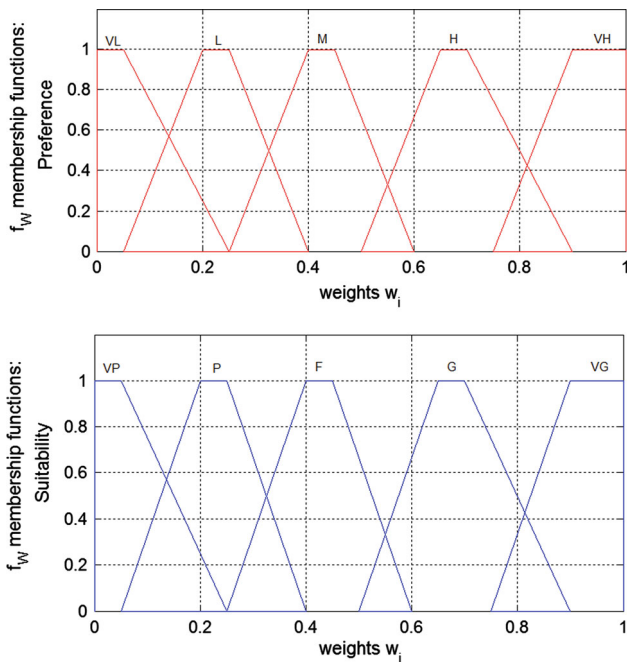


Fig. 7 Membership functions for “Preference” and “Suitability”

tip to the spring pin. The spring pin and the plastic heel are co-injected.

All the alternatives use a solid metallic pin, except for alternatives A4 and A5 which use a hollow pin placed at a specific distance from the cross section.

3 Results

Fuzzy Linguistic Variables are introduced to describe the preferences of the decision makers. The variables used to indicate the “preference” are taken from the Term Set {Very Low (VL), Low (L), Medium (M), High (H), Very High (VH)}. The variables describing the Fuzzy Variable for “suitability”, are taken from the set {Very Poor (VP), Poor (P), Fair (F), Good (G), Very Good (VG)}. Figure 7 shows the membership functions for “Preference” and “Suitability”.

Table 2 Linguistic values for the preferences expressed by the decision makers

	DM1	DM2	DM3
C1	H	L	M
C2	M	L	VH
C3	H	H	H
C4	L	VH	M
C5	VH	H	M
C6	VH	VH	H

3.1 F-AHP

In the following section the application of the F-AHP method to the specific case study is described.

3.1.1 Linguistic values assessed in relation to the preferences of the decision makers

Linguistic values are provided by the decision makers (Table 2), to evaluate the suitability of each alternative in relation to the criteria, and Table 3 reports the corresponding numerical values.

Hence, the Mean Operator is used to find the mean aggregated preference weight W_j as in Eq. (3). The W_j matrix is:

$$W_j = \begin{pmatrix} 0.26 & 0.41 & 0.46 & 0.63 \\ 0.35 & 0.50 & 0.56 & 0.66 \\ 0.50 & 0.65 & 0.70 & 0.90 \\ 0.35 & 0.50 & 0.56 & 0.66 \\ 0.50 & 0.65 & 0.71 & 0.83 \\ 0.66 & 0.81 & 0.90 & 0.96 \end{pmatrix}$$

Decision makers are then required to analyse the suitability of the alternatives by linguistic terms.

If $S_{jmr} = (a_{jmr}, b_{jmr}, c_{jmr}, d_{jmr})$ is defined as the suitability assigned to the m th alternative ($m = 1, 2, \dots, p; p = 6$), evaluated against the j th criterion ($j = 1, 2, \dots, i; i = 5$), by the r th decision maker ($r = 1, 2, \dots, q; q = 3$), the

Table 3 Numerical preference values

DM1				DM2				DM3			
0.50	0.65	0.70	0.90	0.05	0.20	0.25	0.40	0.25	0.40	0.45	0.60
0.25	0.40	0.45	0.60	0.05	0.20	0.25	0.40	0.75	0.90	1	1
0.50	0.65	0.70	0.90	0.50	0.65	0.70	0.90	0.50	0.65	0.70	.90
0.05	0.20	0.25	0.40	0.75	0.90	1	1	0.25	0.40	0.45	0.60
0.75	0.90	1	1	0.50	0.65	0.70	0.90	0.25	0.40	0.45	0.60
0.75	0.90	1	1	0.75	0.90	1	1	0.50	0.65	0.70	0.90

Table 4 Suitability Indices for all the criteria (S_{IndexCj})

$S_{IndexC1} = \begin{pmatrix} 0.266 & 0.416 & 0.466 & 0.633 \\ 0.750 & 0.900 & 1.000 & 1.000 \\ 0.500 & 0.650 & 0.700 & 0.900 \\ 0.750 & 0.900 & 1.000 & 1.000 \\ 0.500 & 0.650 & 0.716 & 0.833 \\ 0.116 & 0.266 & 0.316 & 0.466 \end{pmatrix}$	$S_{IndexC2} = \begin{pmatrix} 0.350 & 0.500 & 0.550 & 0.733 \\ 0.183 & 0.333 & 0.383 & 0.533 \\ 0.166 & 0.266 & 0.316 & 0.483 \\ 0.200 & 0.350 & 0.400 & 0.566 \\ 0.666 & 0.816 & 0.900 & 0.966 \\ 0.016 & 0.066 & 0.116 & 0.300 \end{pmatrix}$
$S_{IndexC3} = \begin{pmatrix} 0.033 & 0.133 & 0.183 & 0.350 \\ 0.116 & 0.266 & 0.316 & 0.466 \\ 0.750 & 0.900 & 1.000 & 1.000 \\ 0.416 & 0.566 & 0.616 & 0.800 \\ 0.666 & 0.816 & 0.900 & 0.966 \\ 0.016 & 0.066 & 0.116 & 0.300 \end{pmatrix}$	$S_{IndexC4} = \begin{pmatrix} 0.033 & 0.133 & 0.183 & 0.350 \\ 0. & 0. & 0.050 & 0.250 \\ 0.750 & 0.900 & 1.000 & 1.000 \\ 0.033 & 0.133 & 0.183 & 0.350 \\ 0.666 & 0.816 & 0.900 & 0.966 \\ 0.583 & 0.733 & 0.800 & 0.933 \end{pmatrix}$
$S_{IndexC5} = \begin{pmatrix} 0.016 & 0.066 & 0.116 & 0.300 \\ 0.116 & 0.266 & 0.316 & 0.466 \\ 0.750 & 0.900 & 1.000 & 1.000 \\ 0.416 & 0.566 & 0.616 & 0.800 \\ 0.666 & 0.816 & 0.900 & 0.966 \\ 0.016 & 0.066 & 0.116 & 0.300 \end{pmatrix}$	$S_{IndexC6} = \begin{pmatrix} 0.500 & 0.650 & 0.700 & 0.900 \\ 0.500 & 0.650 & 0.700 & 0.900 \\ 0.000 & 0.000 & 0.050 & 0.250 \\ 0.033 & 0.133 & 0.183 & 0.350 \\ 0.583 & 0.733 & 0.800 & 0.933 \\ 0.583 & 0.733 & 0.800 & 0.933 \end{pmatrix}$

values of the suitability indices corresponding to the first criterion are:

$$S_{C1} = \begin{pmatrix} (0.25 & 0.40 & 0.45 & 0.60) & (0.50 & 0.65 & 0.70 & 0.90) & (0.05 & 0.20 & 0.25 & 0.40) \\ (0.75 & 0.90 & 1.00 & 1.00) & (0.75 & 0.90 & 1.00 & 1.00) & (0.75 & 0.90 & 1.00 & 1.00) \\ (0.50 & 0.65 & 0.70 & 0.90) & (0.50 & 0.65 & 0.70 & 0.90) & (0.50 & 0.65 & 0.70 & 0.90) \\ (0.75 & 0.90 & 1.00 & 1.00) & (0.75 & 0.90 & 1.00 & 1.00) & (0.75 & 0.90 & 1.00 & 1.00) \\ (0.75 & 0.90 & 1.00 & 1.00) & (0.50 & 0.65 & 0.70 & 0.90) & (0.25 & 0.40 & 0.45 & 0.60) \\ (0.25 & 0.40 & 0.45 & 0.60) & (0.05 & 0.20 & 0.25 & 0.40) & (0.05 & 0.20 & 0.25 & 0.40) \end{pmatrix}$$

$$\times \left(\frac{0.45 + 0.70 + 0.25}{3}, \frac{0.60 + 0.90 + 0.40}{3} \right) = (0.266, 0.416, 0.466, 0.633) \tag{11}$$

These matrices can be collected into a single matrix, with six criteria and six alternatives.

3.1.3 Fuzzy Score calculation

For the final ranking, a Fuzzy Score Γ is calculated as in Eq. (6), for each alternative. For the first alternative, the value of the Fuzzy Score is:

$$\tilde{\Gamma}_1 = \frac{(S_{11} \otimes W_1) \oplus (S_{21} \otimes W_2) \oplus (S_{31} \otimes W_3) \oplus (S_{41} \otimes W_4) \oplus (S_{51} \otimes W_5) \oplus (S_{61} \otimes W_6)}{6} = (0.094 \ 0.192 \ 0.246 \ 0.426)$$

The matrix corresponding to the evaluation of the fuzzy scores, for all the alternatives, is shown in Table 5.

Each fuzzy score is a trapezoidal fuzzy number and consequently can be graphically represented as in Fig. 8.

3.1.4 Defuzzification

The defuzzified values, calculated with the Weighted Average Method [as in Eq. (8)] are collected in Eq. (12), in which

3.1.2 Suitability indices are evaluated for all the criteria

The suitability indices (S_{jm}) are then evaluated for each criterion, as in Eq. (4).

For example, for the first criterion and the first alternative, the suitability index can be calculated as in Eq. (11) which is the first row, of the $m \times 4$ matrices in Table 4, in which the suitability indices for all the alternatives and criteria are depicted.

$$\tilde{S}_{11} = \frac{\sum_{r=1}^q S_{11r}}{q} = \frac{S_{111} \oplus S_{112} \oplus S_{113}}{q} = \left(\frac{0.25 + 0.50 + 0.05}{3}, \frac{0.40 + 0.65 + 0.20}{3} \right)$$

Table 5 Fuzzy Scores for all the alternatives

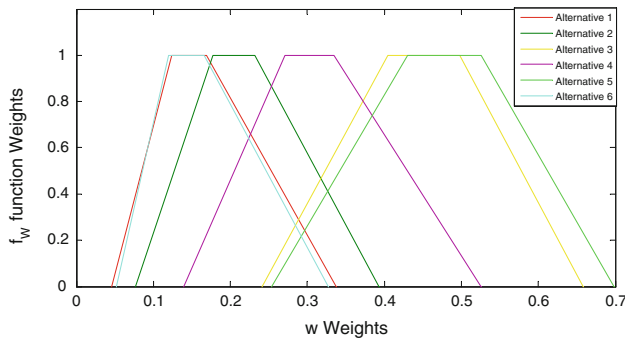
$$\Gamma_{tot} = \begin{pmatrix} \tilde{\Gamma}_1 \\ \tilde{\Gamma}_2 \\ \tilde{\Gamma}_3 \\ \tilde{\Gamma}_4 \\ \tilde{\Gamma}_5 \\ \tilde{\Gamma}_6 \end{pmatrix} = \begin{pmatrix} 0.094 & 0.192 & 0.246 & 0.426 \\ 0.119 & 0.236 & 0.298 & 0.472 \\ 0.200 & 0.337 & 0.422 & 0.589 \\ 0.120 & 0.244 & 0.306 & 0.495 \\ 0.276 & 0.458 & 0.558 & 0.732 \\ 0.108 & 0.199 & 0.259 & 0.423 \end{pmatrix}$$


Fig. 8 Fuzzy scores for the alternatives

each row corresponds to the mean value of the fuzzy score of each alternative.

$$\Gamma_{mean} = \begin{pmatrix} 0.225 \\ 0.272 \\ 0.382 \\ 0.280 \\ 0.507 \\ 0.235 \end{pmatrix} \tag{12}$$

Thus the elements of this vector can be ranked to search for the maximum value among the scores, which corresponds to the best alternative. In this case the fifth alternative is considered as the best, followed by the third and fourth.

3.1.5 Sensitivity analysis

A sensitivity analysis is thus performed to understand how the overall decision is sensitive to small changes applied to the weight judgments given to the criteria. When the definition of the importance of the weights is uncertain, the sensitivity analysis seems to play a key role in selecting concept designs.

Thus 12 experiments were conducted, and the results are reported in Table 6.

Following the example of [31], the weights are set, one at a time, as equal to VL, L, M, H, VH, for the first five experiments. Then, from the sixth to the eleventh experiment, each criterion is set at the highest score of the preference and the others are set to the minimum score. This is done in order to understand which criterion influences the process the most.

Regarding the twelfth experiment, the development cost is set to the minimum score and the remaining ones to the

highest in order to understand how the result is influenced by the cost criterion.

The results are depicted in Fig. 9. In eight of the twelve experiments, A5 achieved the first position (66.7 % of preferences), while in three experiments, A3 was the most preferred. Only in one experiment did the fourth alternative (A4) achieve the highest score.

3.2 PuCC

The same sets of alternatives were adopted for both the PuCC approach and the F-AHP experiment (Fig. 6).

A design concept was added to the set, to provide a basis for comparison during the screening. First the A0 device, which is currently on the market, was chosen as the datum concept (Fig. 5).

As shown in Table 7, three iterations were made, with three different datum concepts: at the first run, the original concept was chosen as the datum and alternatives A2 and A4 were eliminated. In the second run, the best scoring alternative of the previous run (A5), was selected as the new concept datum, while A1 was shown to be a weak design concept for the heel tip device.

In the same run, A3 and A5 had the same ratings.

In the third run, keeping the second best alternative of the previous iteration (A3) as the datum concept, another alternative was eliminated.

The three alternatives A0, A3, A5 were therefore considered for further analyses. As A5 achieved the same score as A3, and A0 reached a “-1” point, a further computation was needed in order to get a more precise result and this involved the criteria weights calculated in the previous step.

In Table 8 the criteria are compared pairwise and the last column reports the priority vector. The reliability of the assembly (C3) seems to rank higher than the minimal modification to the heel assembly (C6), while according to the decision makers manufacturing ease (C4) was—equal to the development cost (C5). Aesthetics (C2) was the least important.

The alternatives which, from previous iterations, have been designated as candidates for the selection process, are thus rated against the given criteria. A maximum possible vote is decided for each alternative (Table 9).

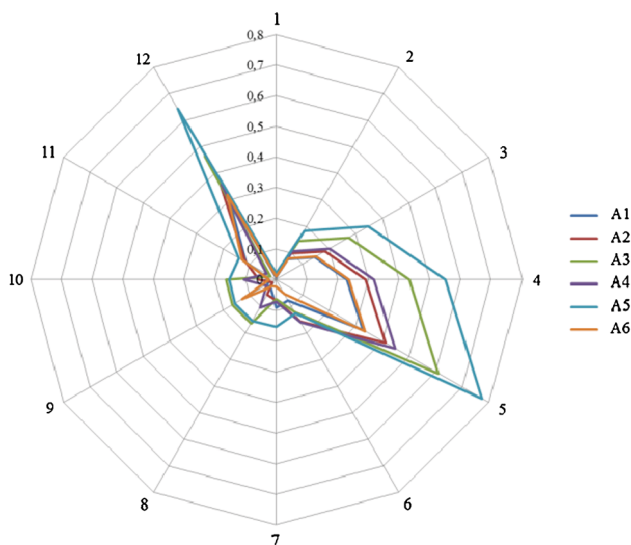
The normalized values of the alternative ratings and the merit functions MF_i are reported in Table 10.

3.2.1 Sensitivity analysis

A sensitivity analysis is performed, as for the F-AHP method, in order to understand how small changes in the importance of certain attributes or in the alternative ratings influence the results of the analysis.

Table 6 Results of the experiments for the sensitivity analysis

Exp. n	Definition	Alternatives						Ranking
		A1	A2	A3	A4	A5	A6	
1	$W_{C1-C6} = VL$	0.009	0.012	0.017	0.013	0.021	0.009	5>3>4>2>6>1
2	$W_{C1-C6} = L$	0.078	0.098	0.145	0.107	0.184	0.079	5>3>4>2>6>1
3	$W_{C1-C6} = M$	0.146	0.184	0.273	0.201	0.347	0.149	5>3>4>2>6>1
4	$W_{C1-C6} = H$	0.231	0.292	0.433	0.319	0.550	0.237	5>3>4>2>6>1
5	$W_{C1-C6} = VH$	0.326	0.412	0.610	0.449	0.775	0.334	5>3>4>2>6>1
6	$W_{C1} = VH; W_{C2-C6} = VL$	0.077	0.158	0.121	0.159	0.127	0.055	4>2>5>3>1>6
7	$W_{C2} = VH; W_{C1,C3-C6} = VL$	0.090	0.067	0.062	0.070	0.154	0.024	5>1>4>2>3>6
8	$W_{C3} = VH; W_{C1-C2,C4-C6} = VL$	0.034	0.057	0.164	0.104	0.154	0.024	3>5>4>2>1>6
9	$W_{C4} = VH; W_{C1-C3,C5-C6} = VL$	0.034	0.015	0.164	0.037	0.154	0.128	3>5>6>4>1>2
10	$W_{C5} = VH; W_{C1-C4,C6} = VL$	0.023	0.057	0.164	0.104	0.154	0.024	3>5>4>2>6>1
11	$W_{C6} = VH; W_{C1-C5} = VL$	0.113	0.116	0.021	0.037	0.140	0.128	5>6>2>1>4>3
12	$W_{C5} = VL; W_{C1-C4,C6} = VH$	0.312	0.367	0.463	0.357	0.643	0.320	5>3>2>4>6>1

**Fig. 9** Results of the sensitivity analysis for the Fuzzy-AHP application

This is particularly relevant for the weights, due to the subjective nature of the judgments and to the fact that the final values the two candidates (A0 and A5) were similar in terms of merit function.

Despite small changes in the weights of the attributes, the solution that maintains the same position in the ranking is said to be stable and, thus, should be selected as the preferred solution. As in [10], two investigations are carried out: the first regards the change in the weights of the attribute values, and the second in the rating of the alternatives.

In the former, the changes are made to the elements of the importance vector, by altering them by +5 % in favour of the second candidate, and then checking whether the original ranking of the alternatives changes. The results shown in

Table 11, highlight that the ranking result is stable and is not affected by the small changes to the attribute weighting.

An additional test is performed to investigate the stability of the solution to small changes in the alternative ratings. Also in this case, the merit function values, in terms of both the increase and the decrease in 5 % of the ratings of the alternatives, did not modify the ranking order of the candidate solutions (see Table 12).

The merit functions of the first alternative rank higher than the second alternative, both with a 5 % decrement and increment in ratings. The solution is thus stable and the corresponding alternative can be selected for subsequent development.

4 Discussion

The comparison strategy described above presents some limitations that are worth highlighting.

Looking at the problem from a general perspective, a campaign of complex case studies needs to be carried out, in order to propose comprehensive conclusions. As mentioned in the introduction, the aim of this paper is to compare the two selection approaches considered (F-AHP and PuCC) on the same case study, rather than obtaining a complete validation of the individual methods. While the validity of both these methods has been widely confirmed in the literature, a comparison of these specific methods, on the same case study, is novel.

The definition of complexity is a debated problem in literature.

As is well explained by Eckert et al. [34], the definition of complexity within the engineering design domain is linked to several aspects. For example, the uncertainty in information

Table 7 Three iterations of the PuCC are performed for screening alternatives: Results: changes in the datum concept (first run: A0; second run: A5; third run: A3); concept designs removed (first run: A2, A4; second run: A1; third run: A6)

	A0			A1			A2			A3			A4			A5			A6		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
C1	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Reduction of the pin height (add volume for wearing material)	-	-	-	+	+	0	0	0	0	+	+	0	0	0	0	+	+	0	0	0	0
Aesthetics	-	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Assembly reliability	0	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Manufacturability	+	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Development cost	-	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Minimal modification to the heel assembly	0	0	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
First datum	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Second datum	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Third datum	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Net score	0	-2	-1	-3	-4	-4	-4	-4	-4	-2	0	0	0	0	0	-4	0	0	0	0	-3

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Table 8 Comparison between criteria and relative importance vector

	C1	C2	C3	C4	C5	C6	Relative importance	Importance vector
C1	1	1	0	0	0	0	2	0.095
C2	0	1	0	0	0	0	1	0.047
C3	1	1	1	1	1	1	6	0.285
C4	1	1	0	1	0.5	0	3.5	0.166
C5	1	1	0	0.5	1	0	3.5	0.166
C6	1	1	0	1	1	1	5	0.238
							Total: 21	

Importance vector (I.V.) Normalized relative importance

Table 9 Ratings of the selected alternatives against each attribute

	A0	A3	A5
C1	0	7	8
C2	7	3	10
C3	8	8	8
C4	8	7	4
C5	9	9	6
C6	10	1	10
MAX-RATING	10	10	10

Table 10 Merit Functions for ranking the alternatives

	A0	A3	A5	Importance vector (I.V.)
C1	0	0.70	0.80	0.095
C2	0.70	0.30	1	0.047
C3	0.80	0.80	0.80	0.285
C4	0.80	0.70	0.40	0.166
C5	0.90	0.90	0.60	0.166
C6	1	0.10	1	0.238
MF_i	0.783	0.600	0.757	
Ranked alternatives	1	3	2	

regarding design features can lead to a difficult prediction of the behavior of the final product. Moreover, in a complex product, where the parts are closely linked, a change to one individual part, results in modifications to the other parts in the assembly.

Other aspects of complexity include a higher number of alternatives/criteria or the presence of interdependencies between the criteria. Several examples of the former aspect have been proposed in the literature. In [27] for example, a tuned gyroscope is selected from 15 alternatives against 18 criteria. Other selection problems, characterized by interdependencies between criteria are often solved by more complex selection methods, such as Saaty's Analytic Network Process (ANP).

Our case study is not characterized by a high level of uncertainty and number of alternatives but the strict set of specifications and constraints, and the close correlation between parts guarantee a not trivial degree of complexity.

In addition, very complex problems represent only a portion of daily industrial practice. Therefore, a comparison strategy might be more effective if it focused on more standard problems, rather than deepening more complex, but unusual, ones. This should result in a simplification and acceleration of the knowledge transfer of selection methods to industrial practitioners.

In terms of a detailed analysis of F-AHP, another limitation is in the choice of suitable preference aggregation functions. In fact, a trade-off strategy between design criteria would be better at describing the engineering design problems. There are two design strategies that perform trade-offs [36]: non-compensatory and compensatory. The non-compensatory strategy is a more conservative approach. It achieves a trade-off between design parameters, to improve the goals that generally are less easy to achieve thereby leading to a better the design. In the compensatory strategy, on the other hand, the priorities of some of the weaker goals can be reduced and are compensated for by a slight increase in other ones. This is a more "aggressive" approach. These two approaches should be considered as alternative preferences during the decision process.

Many authors have dealt with this problem, for example Scott and Antonsson [37] introduced the Method of Imprecision (MoI), based on the mathematics of fuzzy sets, for modelling design preferences in the case of imprecision and uncertainty. This method provides trade-off functions for aggregating decision maker's preferences in an overall one. This trade-off consists of building up several functions into one individual function that describes the overall preferences. This function ranges from the non-compensating *min* function through a partially compensating P_π function, to the totally compensating *max* function.

Other aggregation functions have been considered to represent trade-offs in engineering design, for example *t-norms* and *t-conorms* [38]. Unfortunately, they do not satisfy the

Table 11 Sensitivity of solution to changes in the relative importance of attributes

	A0	A3	A5	Import. (I.V.)	vector	I.V. after sensitivity analysis	Changes
C1	0	0.70	0.80	0.095		0.045	IV−5 %
C2	0.70	0.30	1	0.047		0.098	IV+5 %
C3	0.80	0.80	0.80	0.285		0.236	IV−5 %
C4	0.80	0.70	0.40	0.166		0.217	IV+5 %
C5	0.90	0.90	0.60	0.166		0.217	IV+5 %
C6	1	0.10	1	0.238		0.288	IV+5 %
MF ₁ before sensit. an.	0.783	0.600	0.757				
MF ₁ after sensit. an.	0.913	0.625	0.827				

Table 12 Evaluation of Merit Function, with a 5 % decrease/increase in the ratings of the two best alternatives

	A0		A5	
	5 %dec	5 %inc	5 %dec	5 %inc
C1	0.783	0.788	0.757	0.757
C2	0.783	0.784	0.757	0.757
C3	0.782	0.785	0.756	0.759
C4	0.783	0.784	0.756	0.758
C5	0.783	0.784	0.756	0.758
C6	0.782	0.785	0.756	0.758

axioms for appropriate aggregation functions, such as *annihilation*, *idempotency* and *monotonicity*, as extensively showed in the literature [37].

In this paper we proposed a simpler, but not simplistic, approach for aggregating preferences. Trapezoidal Fuzzy Numbers were chosen to translate verbal judgments into fuzzy numbers. In addition, a weighted sum approach is used to aggregate functions for the overall preference representation. As noted in [37], the weighted sum approach is a compensatory strategy. Thus, the use of this kind of function to aggregate decision makers’ preferences, could provide a partial view of the group’s overall preference.

5 Conclusion

Two selection methods, PuCC and F-AHP were applied to the same design problem. The criteria used to compare the methods are summarized in Table 13.

Regarding the calculation carried out with the PuCC method, inclusion of the benchmark datum concept A0 influences the results. In fact it achieved the maximum score, followed by alternative A5 (Fig. 6), which, in contrast, seems to be the preferred alternative in the F-AHP application.

In the PuCC method however, the values of the merit functions relative to alternatives A0 and A5 were similar (0.783 against 0.757 in Table 11). Alternative A5 can therefore be chosen as the preferred option of the innovative proposals.

Although F-AHP needs more mathematical calculation than PuCC, and thus seems more time-consuming, once the software is written, F-AHP is easier—to use in more applications than the PuCC method.

On the other hand, PuCC is a three-step method, and consequently is very concise. However, since PuCC requires interaction between team members, the time spent in team discussion, which is useful for filling in the scoring matrices, needs to be accounted for.

For the *total computational time* required, we define the time elapsed between the insertion of the first data and the attainment of the final result.

In this specific case (six alternatives and six criteria) the *effective computational time* for FAHP lasts fractions of a second (with an AMD 64 Intel, 2.53 GHz processor). In any case, most time is spent in entering the data, which in terms of quantity, depends on the size of the problem. This time period, which may increase with the number of criteria and alternatives, in our case took under 10 min.

In terms of Pugh’s matrices, the time needed increases considerably. The matrices and the related steps of calculation were reported on a Microsoft Excel® spread sheet. In this case, most of the total computational time is spent on team discussions before each scoring session. In our case, the discussion took about 20 min per run, for a total of almost one hour, for three runs of the matrices.

In the PuCC method, once the process is finished, the screening and scoring matrices can only be partially reprocessed for further problems. In fact each problem provides different results from the team discussions and generates different “hybrid” solutions.

In the screening phase of the PuCC method, the voting procedure is not used, thus the decision makers are allowed

Table 13 Pros and cons of the FAHP and the PuCC methods for selection of concept alternatives

Criteria	Methods			
	F-AHP		PuCC	
	Advantages	Drawbacks	Advantages	Drawbacks
Computational effort	Analytical process → well suited for software implementation	Pairwise comparison takes too much time in the case of several alternatives	Simple mathematical passages	Not suitable for software implementation
Suitability for the early design phases	Verbal judgments are nearer than numbers to how humans reason	Computation is too precise and too mathematical	Simple +/−/0 scores for filling in the matrices Pairwise comparison is easy even with many alternatives and criteria Heuristic thinking and communication are facilitated	–
Suitability for group decision making	Takes accurate account of the individual decision maker's viewpoint	–	Team discussion facilitates communication	The divergences could be not overcome
Ease of application	–	Not intuitive due to its mathematical framework	More intuitive due to a discursive structure	–

to use an “on/off” strategy, which is based on a simple comparison between each alternative and the datum concept. This involves clear judgments, avoiding the use of numerical voting.

Thus a better solution for the voting procedure is to use a fuzzy environment, which is able to better describe the uncertainties of the preliminary design stages.

Pugh's Controlled Convergence method is particularly suitable for a large group of alternatives and criteria, however no control is provided for group decision making. In general, as shown by Frey et al. [23], in Pugh's method, the experts have a previous team discussion, in which any controversies are generally resolved. The experts then draw up Pugh's evaluation matrices and, in the case of persistent disagreements, they enter an “S” (or 0)—which means “similar to” the datum concept—in the corresponding cell of the matrix [23]. In general, when many alternatives need to be compared, the AHP-based methods do not seem convenient, as the alternatives need to be pairwise compared against each criterion, as discussed extensively in the literature.

On the other hand, the analytical framework of the F-AHP makes it suitable for software implementation, resulting in a considerable increase in the speed of the application.

In addition, the choice of a fuzzy-environment is more suitable for pinpointing more precisely and analytically the decisions and viewpoints of the individual decision maker, than with a team discussion.

The question of which decisional method best satisfies the needs of concept selection is hard to answer with just one solution. In our opinion the final choice should be made by practitioners, according to their specific problems. This could be done after an intensive knowledge transfer, from researchers in engineering design and practitioners, in order to enhance a strategic and profitable exchange of information. This could help both designers to enrich their experience and industrial practitioners to increase their familiarity with decision support tools.

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