



# Analytical or Computer-Aided Graphical Methods for Introductory Teaching of Mechanism Kinematics?

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**Abstract.** The paper presents, through an example, two different approaches to introduce mechanism kinematics to first-cycle mechanical engineering students. Analytical solution and traditional graphic methods are considered, both supported by state-of-the-art software tools. Advantages and disadvantages of both approaches from an educational point of view are outlined during example development. Finally a summarizing discussion is proposed.

## 1 Introduction

In Italian universities, almost all first-cycle degrees in mechanical engineering include, usually in their second year, a first course on mechanics of machines within which an introduction of kinematics and statics of mechanisms is presented [1]. Typically, the expected practical learning outcomes are the capability to analyze the kinematic behavior (position, velocity, acceleration) of closed loop mechanisms, the operative understanding of the concepts of mobility and degrees of freedom and the consequent choice of independent variables, and the acquisition of introductory notions regarding configurations and singularities. Starting from the properties of rigid bodies and kinematic pairs, the teaching track usually presents methods for the formulation of kinematic models and for their solution. Then a certain amount of application examples developed in detail, in order to allow students to apprehend both theory and practical issues (see for example [2], chapters 2 and 4 for typical reference material).

Focusing on application examples, that represent a key element of learning, there are many possibilities to implement them, and the choice of the approach drives the software tools used for their development. One possibility is to adopt a mixed algebraic-geometric approach [3], based on a well-known educational tool for teaching math and geometry [4]. Other possibilities are either pure algebraic approaches based on computer algebra tools, or more geometrically oriented solutions, based on CAD systems [5–7]: here, through an example, we will discuss and compare these two approaches, the former supported by Maple software tool [8], the latter by using a constraint based parametric 2D CAD software with embedded constraint solver [9, 10] and available free of charge [11]. The two adopted approaches and tools are widely used to solve engineering problems, so, while learning kinematics, students have also the opportunity to tackle and learn engineering tools within the process of solving

engineering problems. Problem solving by the use of complex modelling tools is a key competence of a young mechanical engineer, as in many cases this kind of skill is needed in actual working situations.

The paper presents an example problem, then its solution by means of the two approaches is discussed in details, with specific reference to difficulties and learning opportunities. Finally conclusions are drawn.

## 2 Example Mechanism

A well-known mechanism is chosen for the discussion, the Roberts straight line linkage, shown in Fig. 1: the mechanism is a four bar, whose geometry requires that links 2 and 3 and segments BP and CP on coupler link 4 have the same length, while the length of frame link 1 is twice the length of the segment BC on link 4. To develop the mechanism model, a reference frame is positioned as in Fig. 1a, with its origin O coinciding with coupler point P when the mechanism is in its symmetric position.

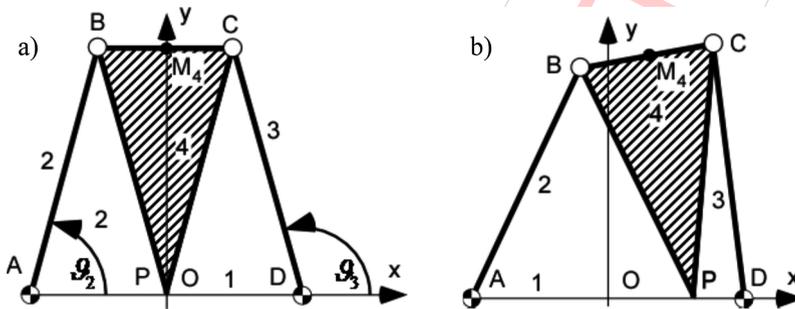


Fig. 1. Roberts straight line linkage

When the mechanism is moved in both directions from its reference symmetric configuration of Fig. 1a, the coupler point P traces a trajectory that for a certain motion range is very close to a straight line coinciding with the  $x$  axis (Fig. 1b).

A typical application problem to be presented to students, after the discussion about fundamentals of kinematic analysis, can be:

- to determine the trajectory  $(x_P, y_P)$  of point P, and consequently the error  $(y_P)$  of the mechanism in tracing a straight line motion
- to determine the angular velocities of the links and the velocity components of point P.

The problem formulation allows students to reason about the difficulties related to closed-loop position analysis, with the determination of the coupler point trajectory and velocity, and, at the same time, to consider the functional aspects of the linkage and how they are related to its dimensional design. In addition, students are led to observe that in kinematics problems approximate solutions are often very useful.

### 3 Solution of Example Mechanism: Analytical Approach

Traditionally, the problem introduced in the previous section can be solved by setting up and solving a mathematical model of the linkage. In order to outline the advantages and disadvantages of this approach from an educational point of view, the problem solution will be developed in the following.

Assuming  $l_2 = l_3 = l_{4BP} = l_{4CP}$  ( $l_2$  hereafter) and  $l_{1AD}/2 = l_{4BC}$  ( $l_4$ ), the linkage closure may be stated by imposing a relation on links 2 and 3 and the length of  $l_4$ :

$$(B - C)^2 - l_4^2 = 0 \quad (1)$$

in which  $B = \begin{bmatrix} -l_4 + l_2 \cos(\vartheta_2) \\ l_2 \sin(\vartheta_2) \end{bmatrix}$  and  $C = \begin{bmatrix} l_4 + l_2 \cos(\vartheta_3) \\ l_2 \sin(\vartheta_3) \end{bmatrix}$

By substituting and simplifying (the use of a computer algebra program such as Maple is very helpful in this activity), the following closure equation, relating angles  $\vartheta_2$  and  $\vartheta_3$ , is obtained:

$$\begin{aligned} (4l_4l_2 - 2l_2^2 \cos(\vartheta_2)) \cos(\vartheta_3) + (-2l_2^2 \sin(\vartheta_2)) \sin(\vartheta_3) \\ + (-4l_4l_2 \cos(\vartheta_2) + 3l_4^2 + 2l_2^2) = 0 \end{aligned} \quad (2)$$

Equation (2) can be rewritten in the well-known form:

$$U(\vartheta_2) \cos(\vartheta_3) + V(\vartheta_2) \sin(\vartheta_3) + W(\vartheta_2) = 0 \quad (3)$$

and it can be solved by the expression [2, App. A]:

$$\vartheta_3 = 2 \arctan \left( \frac{V \pm \sqrt{V^2 + U^2 - W^2}}{U - W} \right) \quad (4)$$

Substitution into Eq. 4 of explicit expressions of coefficients U, V and W (see Eq. 2) yields a complicated and error prone expression, so, in order to obtain numerical results and plots, students should be suggested to adopt either a computer algebra tool (e.g., Maple) or to formulate the problem with an algorithmic approach (Matlab, Excel) allowing them to compute numerical results from Eq. (4) through a step by step method.

Once angle  $\vartheta_3$  is obtained (for the considered mechanism by adopting the “+” solution of Eq. (4)), angle  $\vartheta_4$  can be obtained by the following relation:

$$\vartheta_4 = \arctan 2((C_y - B_y), (C_x - B_x)) \quad (5)$$

yielding:

$$\vartheta_4 = \arctan 2((l_2 \sin(\vartheta_3) - l_2 \sin(\vartheta_2)), (2l_4 + l_2 \cos(\vartheta_3) - l_2 \cos(\vartheta_2))) \quad (6)$$

Finally, the coordinates of point P as a function of angles  $\vartheta_2$  and  $\vartheta_4(\vartheta_2)$  are determined (point  $M_4$  is the midpoint of segment BC,  $l_{4h} = \sqrt{l_2^2 - (l_4/2)^2}$  is the length of segment  $M_4P$ , Fig. 1):

$$P = M_4 + (P - M_4) = \frac{(C + B)}{2} + l_{4h} \begin{pmatrix} \sin(\vartheta_4) \\ -\cos(\vartheta_4) \end{pmatrix} \quad (7)$$

Again, the explicit substitution of  $\vartheta_3$  solution into Eqs. (5), (6) and (7) is very impractical, so resorting to some form of software tool and implementation is required.

Having assumed  $l_2 = 180$  mm and  $l_4 = 120$  mm, the plots in Fig. 2, obtained by Maple, show some numerical results of position analysis: (a) the angle  $180 - \vartheta_3$  whose behavior is symmetrical to that of  $\vartheta_2$ , the two angles being equal in the reference configuration of Fig. 1a, (b) the trajectory  $(x_p, y_p)$  of coupler point P, in which the y coordinate represents the error with respect to exact straight line tracing; please note that axes in Fig. 2b have very different scales, so a student could appreciate that the tracing is approximate, but with an error lower than 1 mm for a range  $-130 \text{ mm} < x_p < 130 \text{ mm}$ .

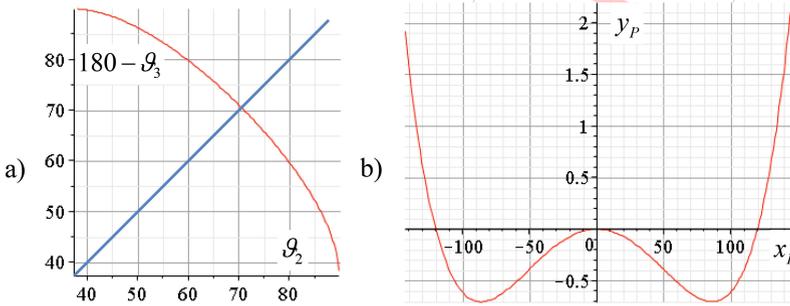


Fig. 2. Plots of  $180 - \vartheta_3$  (deg) and  $(x_p, y_p)$  (mm)

The velocity equation to determine  $\omega_3$  can be easily obtained by deriving Eq. 2, while  $\omega_4$  is less easily obtained either by deriving Eq. 6 or by solving the first system in Eq. 8 (see next Section); here, thanks to Maple features, the direct differentiation is adopted. Plots in Fig. 3 show  $\omega_3$  and  $\omega_4$  as functions of  $\vartheta_2$ , for  $\omega_2 = 1$  rad/s.

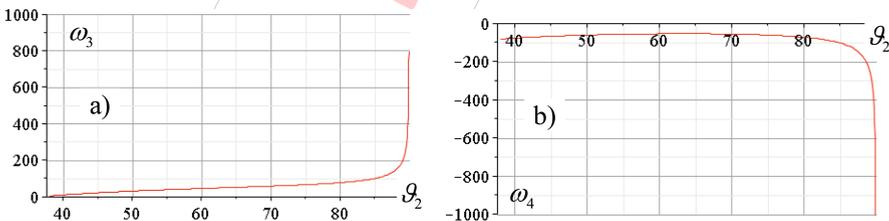


Fig. 3. Plots of  $\omega_3$  and  $\omega_4$  (deg/s)

## 4 Solution of Example Mechanism: Computer-Aided Graphical Method

Before computational approaches were widely adopted due to the increasing availability of suitable computational tools, graphics methods were generally used to study mechanism kinematics. The main advantages of such approaches are the relative simplicity and the clear geometrical meaning and insight, both features being very valuable for introductory educational steps. Unfortunately, one key aspect of graphic methods is that “hand-made” graphic constructions or even traditional CAD based approaches provide only static pictures of the mechanism position and velocities, so that many drawings are necessary to get an overall comprehension of the mechanism characteristics. To overcome this drawback, without resorting to complex 3D modelling tools that may not be suited to second-year students, an interesting solution is provided by 2D parametric CAD tools equipped with embedded constraint solvers, as briefly discussed in Introduction. Almost all parametric 3D software products (e.g., PTC Creo Parametric, Dassault Catia, Siemens NX, Dassault Solidworks, Siemens Solidedge) contain a software module, here generically named “sketcher”, devoted to the sketching of 2D sections, shapes, curves and other planar geometric entities useful to build complex 3D geometries according to a constructive approach. All parametric sketchers share some interesting characteristics, also useful in graphical methods for planar mechanism kinematics and statics:

- it is possible to apply geometric constraints to the sketched objects; such constraints (e.g., horizontal, vertical, coincident, tangent, orthogonal, parallel,...) are permanently enforced and constrain the sketch variations; in other terms, they build up the constraint manifold of the mechanism;
- similarly, it is possible to add dimensional constraints, i.e., “driving dimensions” (e.g., lengths, angles, radii, equality between dimensions,...) whose values determine the sketch size; moreover, the sketcher allows to add “driven dimensions”, i.e. dimensions that do not constrain the sketch, but are useful to measure quantities as a function of the driving dimensions;
- the sketcher computes the number of degrees of freedom of the sketch on the basis of imposed geometric and dimensional constraints, indicating when the sketch is fully determined (zero d.o.f.);
- the sketcher recognizes conflicts among constraints and (usually) asks the user to choose a set of non-conflicting, independent constraints.

Although sketcher software modules are nowadays a well-established component of several 3D modelling tools, it is less easy to find stand-alone 2D software tools with the previous features: here we use the product “Solid Edge 2D Drafting” from Siemens [11], which is a 2D subset of the 3D Solid Edge. For our educational purposes it is relevant the fact that Solid Edge 2D is freely distributed by Siemens. Moreover Solid Edge 2D, beyond fully supporting parametric and constraint based functions, is also a featured 2D engineering drawing tool.

In order to create the model of the chosen mechanism within Solid Edge 2D, a rough sketch is first created, then dimensional and geometric constraints are added, and

finally driven dimensions (e.g., those required to measure the position  $x, y$  of point P). Figure 4a shows the Roberts mechanism in its reference position with three driving dimensions ( $l_1, l_2$  and  $l_{4BC}$ , drawn in black), equality constraints imposed so that  $l_2 = l_3 = l_{4BP} = l_{4CP}$  and various driven dimensions (drawn in light blue in figure), in particular the angles  $\vartheta_2$  and  $180 - \vartheta_3$ , and the two coordinates of P. According to its constraints, the sketch in Fig. 4a has one d.o.f., so it can be moved to  $\infty^1$  different positions, for example by dragging the vertical green segment, connected to point P, that was purposely created as a handle to move the mechanism. Figure 4-b shows the mechanism in the position with  $x_P = 50$  mm; the configuration has been obtained by transforming the driven dimension  $x_P$  (Fig. 4a) into a driving one and assigning to it the value 50. This change eliminates the remaining d.o.f., so the sketch in Fig. 4b is geometrically determined. Such possibility of switching back and forth the status of dimensions between driving and driven is very useful for educational purposes, as it allows student to become acquainted to the concepts of degrees of freedom and free coordinates; moreover, it allows to freely change the independent variables of the mechanism. This kind of flexibility is much more complex to be obtained with analytical approaches, as a change of the independent variables requires to determine a new solution of the mathematical model.

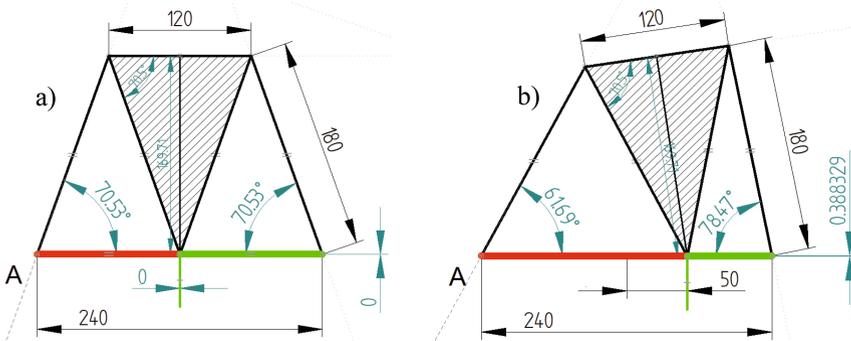
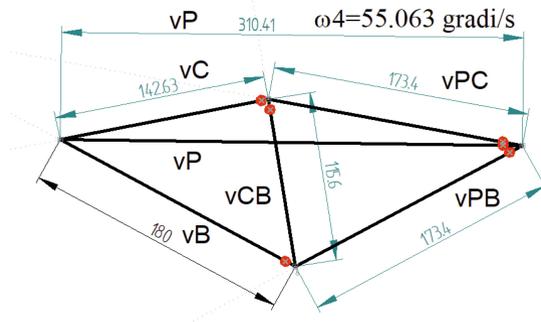


Fig. 4. Parametric sketch of Robert linkage (Color figure online)

A second sketch (Fig. 5, the red circles indicate vector arrows, velocities are expressed in mm/s), related to the position one, can be drawn to solve the velocity analysis problem according to the following relations:

$$\begin{cases} \underline{v}_B = \underline{\omega}_2 \times (B - A) \\ \underline{v}_C = \underline{v}_B + \underline{\omega}_3 \times (C - B) = \underline{v}_B + \underline{v}_{CB} \\ \underline{v}_C = \underline{\omega}_3 \times (C - D) \end{cases} \quad (8)$$

$$\begin{cases} \underline{v}_P = \underline{v}_B + \underline{\omega}_4 \times (P - B) = \underline{v}_B + \underline{v}_{PB} \\ \underline{v}_P = \underline{v}_C + \underline{\omega}_4 \times (P - C) = \underline{v}_C + \underline{v}_{PC} \end{cases}$$



**Fig. 5.** Parametric sketch for graphic solution of velocity analysis (mm/s) (Color figure online)

The drawing in Fig. 5 corresponds to the position in Fig. 4b, with velocities computed for an input velocity  $\omega_2 = 1$  rad/s. Geometric constraints between position and velocity sketches assure that the velocity sketch follows the mechanism position; for example, in Fig. 5, the imposed constraints are:  $v_B$  orthogonal to link 2,  $v_{CB}$  orthogonal to segment CB,  $v_C$  orthogonal to link 3, and so on. For 1 d.o.f. mechanisms, if the correct set of constraints is imposed, the velocity sketch becomes dependent on only one driving dimension, the velocity  $v_B = 180$  mm/s in our case. On the basis of velocity polygon in Fig. 5, also the modules of angular velocities can be obtained by explicit relations added to the sketch: here, for example,  $\omega_4$  is obtained as  $v_{PC}/l_{4PC}$  and then displayed in the sketch with a variable embedded in a field text. Unfortunately, since dimensions in sketches are in most cases computed as positive numbers, a more complicated approach is required to determine angular velocity signs.

## 5 Discussion

The two considered approaches to solve the mechanism kinematics have different features that must be evaluated bearing in mind their learning goals:

- *complexity*: the analytical formulation of the problem (Eqs. 1 and 2) is rather straightforward, but its solution, even if it does not require sophisticated mathematical skills (just the knowledge of Eqs. 3 and 4), is moderately cumbersome and there is the risk that students focus on low value algebraic manipulation activities rather than on understanding the key aspects of the mechanical problem; on the other hand, once the basic elements of the selected 2D CAD tool have been learned, the position sketch is usually very simple; a clear comprehension of the velocity problem is required in both approaches;
- *availability of results*: each of the two approaches is suited to best provide specific results; for example, the plots in Figs. 2 and 3 are rather easily created within Maple, provided that the model is solved, while they would require some complicated effort in Solid Edge; on the other hand, interactive graphic restitution of the mechanism motion and velocity vectors, easily obtained in Solid Edge, would be difficultly obtainable in Maple. Another relevant aspect, already mentioned, is that

reversing mechanism input and output variables is simple in Solid Edge, while very costly with the other approach.

In conclusion, from our experience, the graphic approach is best suited to teach introductory mechanism kinematics, but the development of the analytical point of view and the reciprocal validation of the two models may greatly enrich the student experience.

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